[Problems on GEOMETRIC PROGRESSION]

1. Sum to n-terms: (i) 6 + 66 + 666 + … [IIT'74] (ii) ·7 + ·77 + ·777 + …
(iii) 5 + 555 + 55555 + … [JEE'91]

2. *In a G.P. (m+n)-th term be p and (m – n)-th term be q, then prove that its m-th term is $\sqrt{pq}$.

3. If S be the sum, P the product and R the sum of the reciprocals of n terms of a G.P., prove that $\left(\frac{S}{R}\right)^n = P^2$.

4. If the sum of the first n terms of a G.P. is p and the sum of the first 2n terms is 3p, then prove that, the sum of the first 3n terms is 7p.

HINTS : $\frac{a(r^n-1)}{r-1} = p$ $\quad \frac{a(r^{2n}-1)}{r-1} = 3p$ $\quad \frac{a(r^{3n}-1)}{r-1} = 7p$

$$\frac{a(r^n-1)(r^{2n}+r^n+1)}{r-1} = p.(r^{2n}+r^n+1) = p.(4+2+1) = 7p$$

$$\frac{a(r^n-1)}{r-1}(r^n+1) = 3p \qquad \Rightarrow \cancel{p}r^n = 2\cancel{p}$$

5. The common ratio of a G.P. of n–terms is 'r'. S_m is the sum of first m–terms of the series. Prove that, the sum of the pair wise product of the terms is $\frac{r}{r+1}.S_m.S_{m-1}$ [JEE'88]

6. If each of A.M of m & n and G. M of a & b is $\frac{ma+nb}{m+n}$. Express each of m & n in terms of a & b.

7. Divide 42 in three parts such that their product is 512 and they will form a G.P.

8. A G.P consists of an even number of terms. The sum of all the terms is three-times that of the odd terms. Find the common ratio of the G.P. [Ans: r = 2]
[HINTS: By question $S_{2n} = 3[T_1 + T_3 + T_5 + \ldots + T_{2n-1}]$]

9. Determine three numbers 'a', 'b', 'c' in between 2 and 18 such that, (i) a + b + c = 25, (ii) 2, a, b will be three consecutive terms of an A.P and (iii) b, c, 18 are three consecutive terms of an G.P. [IIT'83]

10. *For what value of n , $\frac{a^{n+1}+b^{n+1}}{a^n+b^n}$ is the G.M of a and b ?

11. **If a, b, c, …., l are in G.P, then prove that, $a+b+\cdots+l = \frac{bl-a^2}{b-a}$.

12. *Sum to n-terms: $(x + y) + (x^2 + xy + y^2) + (x^3 + x^2y + xy^2 + y^3) + \ldots\ldots\ldots$

13. One side of an equilateral triangle is 24 cms. The mid-points of its sides are joined to form another triangle. Again another triangle is formed by joining the mid points of the sides of this triangle and the process is continued indefinitely. Determine the sum of the perimeters of all such triangles. [Ans: 144]

14. If $1 + x + x^2 + \cdots\cdots + x^n = (1 + x)(1 + x^2)(1 + x^4)(1 + x^8)$, find n.

15. Express the recurring decimal $\cdot\dot{1}2\dot{5}$ as a rational number.

16. ***Prove that $\underbrace{(666.....6)^2}_{n\text{ digits}} + \underbrace{888.....8}_{n\text{ digits}} = \underbrace{444.......4}_{2n\text{ digits}}$ [JEE’99]

[HINTS: $\text{LHS} = \underbrace{(666.....6)^2}_{n\text{ digits}} + \underbrace{888.....8}_{n\text{ digits}}$

$$\frac{\cancel{6^2}\,4}{\cancel{9^2}\,9}\left(10^n-1\right)^2+\frac{8}{9}\left(10^n-1\right)=\frac{4}{9}\left(10^n-1\right)\left[10^n+1\right]\frac{4}{9}\left(10^{2n}-1\right)=\underbrace{444.......4}_{2n\text{ digits}}$$]

17. ***Determine whether the number $\underbrace{1\,1\,11\cdots1}_{91\text{ digits}}$ is prime or composite [মৌলিক অথবা যৌগিক] .

18. **“The numbers 49, 4489, 444889, ········ obtained by inserting 48 into the middle of the preceding numbers are square of integers”. Is the statement true ? Give reasons. [JEE’91]

19. A, B & C have Rs 5700 in all. The amount of share of them are in G.P. If B had Rs 150 more then their share would be in A.P. Determine the amount of share of each of them. [JEE’90]

20. **If a, b, c are in G.P and $\log_c a$, $\log_b c$, $\log_a b$ are in A.P, prove that the common difference of the progression is $\frac{3}{2}$. [JEE’89]

21. If a, b, c be respectively the the p^{th}, q^{th} and r^{th} term of a G.P., then prove that $a^{q-r}.b^{r-p}.c^{p-q}=1$.

22. If a, b, c be in A.P. and x, y, z in G.P., prove that $x^{b-c}.y^{c-a}.z^{a-b}=1$.

23. If a, b, c, d are in G.P. then prove that

i) a + b, b + c, c + d are in G.P.

ii) $(a-b)^2, (b-c)^2, (c-d)^2$ are in G.P.

iii) $(a^2+b^2), (b^2+c^2), (c^2+d^2)$, are in G.P.

iv) $\frac{1}{a^2+b^2}, \frac{1}{b^2+c^2}, \frac{1}{c^2+d^2}$ are in G.P.

24. If a, b, c, are in G.P. then prove that

(i) $\frac{a^2+ab+b^2}{bc+ca+ab}=\frac{a+b}{b+c}$ and

(ii) $\frac{1}{a+b}, \frac{1}{2b}, \frac{1}{b+c}$ are in A.P.

25. a, b, c are in G.P. and x, y are the A.M.s of a, b and b, c respectively. Show that, $\frac{a}{x}+\frac{c}{y}=2, \frac{1}{x}+\frac{1}{y}=\frac{2}{b}$.

26. If $(2r+1).2^r$ is the r^{th} term of a progression, then find the sum of first n terms of the series.

[HINTS : $t_r = (2r+1).2^r \quad S_n = t_1 + t_2 + t_3 + \ldots + t_n$

$$S_n = 3.2^1 + 5.2^2 + 7.2^3 + \ldots + (2n+1).2^n$$

$$2S_n = \quad 3.2^2 + 5.2^3 + 7.24 + \ldots + (2n-1).2^n + (2n+1).2^{n+1}$$

$$-S_n = 3.2^1 + 2.2^2 + 2.2^3 + \ldots + 2.2^n - (2n+1).2^{n+1}$$

$$S_n = (2n+1).2^{n+1} - 3.2^1 - \left[2^3 + 2^4 + \ldots + 2^{n+1}\right] = (2n-1).2^{n+1} + 2$$]

☞ Future is not a gift, it’s an achievement.

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